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# CP violation as a probe of flavor origin in Supersymmetry

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## Abstract

We address the question of the relation between supersymmetry breaking and the origin of flavor in the context of CP violating phenomena. We prove that, in the absence of the Cabibbo–Kobayashi–Maskawa phase, a general Minimal Supersymmetric Standard Model with all possible phases in the soft–breaking terms, but no new flavor structure beyond the usual Yukawa matrices, can never give a sizeable contribution to $\varepsilon_K$, $\varepsilon'/\varepsilon$ or hadronic $B^0$ CP asymmetries. Observation of supersymmetric contributions to CP asymmetries in B decays would hint at a non–flavor blind mechanism of supersymmetry breaking.

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In the near future, new experimental information on CP violation will be available. Not only the new $B$ factories will start measuring CP violation effects in $B^0$ CP asymmetries, but also the experimental sensitivity to the electric dipole moment (EDM) of the neutron and the electron will be substantially improved. These experiments may provide the first sign of physics beyond the Standard Model.

If new results do appear and we interpret them in the context of Supersymmetry, both experiments have very different implications on the structure of the soft–breaking terms at the supersymmetry breaking scale. The finding of a non–zero EDM for the neutron would simply indicate the presence of new non–negligible flavor independent susy phases [1]. However, a new result in the non–leptonic $B^0$ CP asymmetries would be a direct prove of the existence of a completely new flavor structure in the soft–breaking terms. We can rephrase this sentence in the form of a strict no–go theorem: **"In the absence of the Cabibbo–Kobayashi–Maskawa (CKM) phase, a general Minimal Supersymmetric Standard Model (MSSM) with possible phases in the soft–breaking terms, but no new flavor structure beyond the usual Yukawa matrices, can never give a sizeable contribution to $\varepsilon_K$, $\varepsilon'/\varepsilon$ or hadronic $B^0$ CP asymmetries"**.

Let us first analyze in more detail this strong statement. Indeed, we are going to show that these contributions are at least two orders of magnitude smaller that the required experimental values of $\varepsilon_K$, $\varepsilon'/\varepsilon$, or, in the case of $B^0$ CP asymmetries, the expected experimental sensitivity. Moreover, we always take a vanishing phase in the CKM matrix, i.e. $\delta_{CKM} = 0$, as a way to isolate the effects of the new supersymmetric phases. We do not include in this no–go theorem other CP violation experiments in rare $B$ decays, as for instance $b \to s\gamma$, where the contribution from chirality changing operators is important (see discussion below). This "theorem" applies to any MSSM, i.e. with the minimal supersymmetric particle content, and general **complex** soft–breaking terms, but with a flavor structure strictly given by the two familiar Yukawa matrices or any matrix strictly proportional to them. In these conditions the most general allowed structure of the soft–breaking terms at the large scale, that we call $M_{GUT}$, is,

$$\begin{gathered}
(m_Q^2)_{ij} = m_Q^2\ \delta_{ij} \quad (m_U^2)_{ij} = m_U^2\ \delta_{ij} \quad (m_D^2)_{ij} = m_D^2\ \delta_{ij} \\
(m_L^2)_{ij} = m_L^2\ \delta_{ij} \quad (m_E^2)_{ij} = m_E^2\ \delta_{ij} \qquad m_{H_1}^2 \qquad m_{H_2}^2 \\
m_{\tilde{g}} e^{i\varphi_3} \quad m_{\tilde{W}} e^{i\varphi_2} \quad m_{\tilde{B}} e^{i\varphi_1} \\
(A_U)_{ij} = A_U e^{i\varphi_{A_U}} (Y_U)_{ij} \quad (A_D)_{ij} = A_D e^{i\varphi_{A_D}} (Y_D)_{ij} \quad (A_E)_{ij} = A_E e^{i\varphi_{A_E}} (Y_E)_{ij}.
\end{gathered} \tag{1}$$

where all the allowed phases are explicitly written and one of them can be removed by an R–rotation. All other numbers or matrices in this equation are always real. Notice that this structure covers, not only the Constrained MSSM (CMSSM) [2], but also most of Type I string motivated models considered so far [3,4], gauge mediated models [5], minimal effective supersymmetry models [6–8], etc. However, as recently emphasized [9], as soon as one introduces some new flavor structure in the soft Susy–breaking sector, even if the CP violating phases are flavor independent, it is indeed possible to get sizeable CP contribution for large Susy phases and $\delta_{CKM} = 0$.

Experiments of CP violation in the $K$ or $B$ systems only involve supersymmetric particles as virtual particles in the loops. This means that the phases in the soft–breaking terms can only appear though the mass matrices of the susy particles.

The key point in our discussion is the absence of any new flavor structure, and its role in the low–energy sparticle mass matrices. Once you have any susy phase that can generate CP violation effects the flavor–change will be necessarily given by a product of Yukawa elements. This fact is completely independent of the presence of only one phase or the 5 phases in Eq. (1) plus the additional $\mu$ phase. It is well–known that the Yukawa Renormalization Group Evolution (RGE) is completely independent of all soft–breaking terms [10]. In fact, we can solve the Yukawa RGEs for a given value of $\tan\beta$ independently of all soft–breaking terms, and the size of all Yukawa matrix elements does not change more than a factor $2-3$ from the electroweak scale to the string or susy breaking scale. Then, a typical estimate for the element $(i,j)$ in the $L$–$L$ down squark mass matrix at the electroweak scale would necessarily be (see [2] for details),

$$({m^2}^{(D)}_{LL})_{ij} \approx m_Q^2 \; Y^u_{ik} Y^{u\,*}_{jk} \tag{2}$$

The presence of imaginary parts is a slightly more delicate issue, though, in any case Eq. (2) will always be an approximate upper bound. As explained in [2,10], the RGE equations of all soft–breaking terms are a set of linear differential equations, and thus can be solved as a linear function of the initial conditions,

$$\begin{aligned} m_Q^2(M_W) = & \sum_i \eta_Q^{(\phi_i)} m^2_{\phi_i} + \sum_{i\neq j} \left( \eta_Q^{(g_i g_j)} e^{i(\varphi_i - \varphi_j)} + \eta_Q^{(g_i g_j)\,T} e^{-i(\varphi_i - \varphi_j)} \right) m_{g_i} m_{g_j} \\ & + \sum_i \eta_Q^{(g_i)} \; m^2_{g_i} + \sum_{ij} \left( \eta_Q^{(g_i A_j)} e^{i(\varphi_i - \varphi_{A_j})} + \eta_Q^{(g_i A_j)\,T} e^{-i(\varphi_i - \varphi_{A_j})} \right) m_{g_i} A_j \\ & + \sum_i \eta_Q^{(A_i)} A_i^2 + \sum_{i\neq j} \left( \eta_Q^{(A_i A_j)} e^{i(\varphi_{A_i} - \varphi_{A_j})} + \eta_Q^{(A_i A_j)\,T} e^{-i(\varphi_{A_i} - \varphi_{A_j})} \right) A_i A_j \end{aligned} \tag{3}$$

where $\phi_i$ refers to any scalar, $g_i$ to the different gauginos, $A_i$ to any tri–linear coupling and the different $\eta$ matrices are $3\times 3$ matrices, **strictly real**. In this equation all the allowed phases have been explicitly written. Regarding the imaginary parts, we can see from Eq. (3) that any imaginary part will always be associated to the non–symmetric part of the $\eta_Q^{(g_i g_j)}$, $\eta_Q^{(A_i A_j)}$ or $\eta_Q^{(g_i A_j)}$ matrices independently of the presence of a single phase or an arbitrary number of them in the initial conditions. This is always true in our general framework, and hence the need of large non–symmetric parts in these matrices on the top of large phases is very clear. To estimate the size of these anti–symmetric parts, we can go to the RGE equations for the scalar mass matrices, where we use the same conventions and notation as in [2,10]. Taking advantage of the linearity of these equations we can directly write the evolution of the anti–symmetric parts, $\hat{m}_Q^2 = m_Q^2 - (m_Q^2)^T$ as,

$$\begin{aligned} \frac{d\hat{m}_Q^2}{dt} = & - [\tfrac{1}{2}(\tilde{Y}_U \tilde{Y}_U^\dagger + \tilde{Y}_D \tilde{Y}_D^\dagger)\hat{m}_Q^2 + \tfrac{1}{2}\hat{m}_Q^2(\tilde{Y}_U \tilde{Y}_U^\dagger + \tilde{Y}_D \tilde{Y}_D^\dagger) + 2\; i \; \Im\{\tilde{A}_U \tilde{A}_U^\dagger + \tilde{A}_D \tilde{A}_D^\dagger\} + \\ & \tilde{Y}_U \hat{m}_U^2 \tilde{Y}_U^\dagger + \tilde{Y}_D \hat{m}_D^2 \tilde{Y}_D^\dagger] \end{aligned} \tag{4}$$

where, due to the reality of Yukawa matrices, we have used $Y^T = Y^\dagger$, and following [10] a tilde over the couplings ($\tilde{Y}$, $\tilde{A}$, ...) denotes a re–scaling by a factor $1/(4\pi)$. In the evolution of the $R$–$R$ squark mass matrices, $m_U^2$ and $m_D^2$, only one of the two Yukawa matrices, the one with equal isospin to the squarks, is directly involved. Then, it is easy to understand that these matrices are in a very good approximation diagonal in the SCKM basis once you start with the initial conditions given in Eq. (1). Hence, we can safely neglect the last two terms in Eq. (4) and forget about $\hat{m}^2_U$ and $\hat{m}^2_D$. From Eq. (1), the initial conditions for these anti–symmetric parts at $M_{GUT}$ are identically zero. So, the only source for $\hat{m}^2_Q$ is necessarily $\Im\{A_U A_U^\dagger + A_D A_D^\dagger\}$. Now, we can analyze the RGE for $A_U$,

$$\frac{d\tilde{A}_U}{dt} = \frac{1}{2}\left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{1}{9}\tilde{\alpha}_1\right)\tilde{A}_U - \left(\frac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \frac{1}{9}\tilde{\alpha}_1 M_1\right)\tilde{Y}_U - \tag{5}$$
$$\left(2\tilde{A}_U\tilde{Y}_U^\dagger\tilde{Y}_U + 3Tr(\tilde{A}_U\tilde{Y}_U^\dagger)\tilde{Y}_U + \frac{5}{2}\tilde{Y}_U\tilde{Y}_U^\dagger\tilde{A}_U + \frac{3}{2}Tr(\tilde{Y}_U\tilde{Y}_U^\dagger)\tilde{A}_U + \tilde{A}_D\tilde{Y}_D^\dagger\tilde{Y}_U + \frac{1}{2}\tilde{Y}_D\tilde{Y}_D^\dagger\tilde{A}_U\right)$$

with an equivalent equation for $A_D$. It is clear that given the general initial conditions in Eq. (1), $A_U$ is complex at any scale. However, we are interested in the imaginary parts of $A_U A_U^\dagger$. At $M_{GUT}$ this combination is exactly real, but this is not true any more at a different scale. From Eq. (5), we can immediately see that these imaginary parts are extremely small. Let us, for a moment, neglect the terms involving $\tilde{A}_D\tilde{Y}_D^\dagger$ or $\tilde{Y}_D\tilde{Y}_D^\dagger$ from the above equation. Then, the only flavor structure appearing in Eq. (5) at $M_{GUT}$ is $Y_U$. We can always go to the basis where $Y_U$ is diagonal and then we will have $A_U$ exactly diagonal at any scale. In particular this means that $\Im\{A_U A_U^\dagger\}$ would always exactly vanish. The same reasoning applies to $A_D$ and $\Im\{A_D A_D^\dagger\}$. Hence, simply taking into account the flavor structure, our conclusion is that any non–vanishing element of $\Im[A_U A_U^\dagger + A_D A_D^\dagger]$ and hence of $\hat{m}^2_Q$ must be necessarily proportional to $(\tilde{Y}_D\tilde{Y}_D^\dagger\tilde{Y}_U\tilde{Y}_U^\dagger - H.C.)$. So, we can expect them to be,

$$(\hat{m}^2_Q)_{i<j} \approx K\left(Y_D Y_D^\dagger Y_U Y_U^\dagger - H.C.\right)_{i<j} \longrightarrow (\hat{m}^2_Q)_{12} \approx K\cos^{-2}\beta\ (h_s h_t \lambda^5)$$
$$(\hat{m}^2_Q)_{13} \approx K\cos^{-2}\beta\ (h_b h_t \lambda^3) \qquad (\hat{m}^2_Q)_{23} \approx K\cos^{-2}\beta\ (h_b h_t \lambda^2) \tag{6}$$

where $h_i = m_i^2/v^2$, with $v = \sqrt{v_1^2 + v_2^2}$ the vacuum expectation value of the Higgs, $\lambda = \sin\theta_c$ and $K$ is a proportionality constant that includes the effects of the running from $M_{GUT}$ to $M_W$. To estimate this constant we have to keep in mind that the imaginary parts of $A_U A_U^\dagger$ are generated through the RGE running and then these imaginary parts generate $\hat{m}^2_Q$ as a second order effect. This means that roughly $K \simeq \mathcal{O}(10^{-2})$ times a combination of initial conditions as in Eq. (3). So, we estimate these matrix elements to be $(\cos^{-2}\beta\{10^{-12}, 6\times10^{-8}, 3\times10^{-7}\})$ times initial conditions. This was exactly the result we found for the $A$–$g$ terms in [2]. In fact, now it is clear that this is the same for all the terms in Eq. (3), $g_i$–$A_j$, $g_i$–$g_j$ and $A_i$–$A_j$, irrespectively of the presence of an arbitrary number of new phases.

As we have already said before, the situation in the $R$–$R$ matrices is clearly worse because the RGE of these matrices involves only the corresponding Yukawa matrix and hence, in the SCKM, they are always diagonal and real in extremely good approximation.

Hence, so far, we have shown that the $L$–$L$ or $R$–$R$ squark mass matrices are still essentially real.

The only complex matrices, then, will still be the $L$–$R$ matrices that include, from the very beginning, the phases $\varphi_{A_i}$ and $\varphi_\mu$. Once more, the size of these entries is determined by the Yukawa elements with these two phases providing the complex structure. However, this situation is not new for these more general MSSM models and it was already present even in the CMSSM.

From here we can start the analysis of the effects of supersymmetric phases in the CP observables. We have already seen that the structure of the sfermion mass matrices remains the same as in the CMSSM case. This is simply due to our dependence to the Yukawa matrices to get any flavor change. On the other hand, the new gaugino phases enter the chargino and neutralino mass matrices. However, in all our previous works [2,6] we have always ignored the EDM bounds, which means that $\varphi_\mu$ could take any value and large phases in the mixing matrices were already present. So, the inclusion of the new gaugino phases does not lead to new effects apart from those already accounted for varying $\varphi_\mu$.

In first place, we will consider indirect CP violation both in the $K$ and $B$ systems, refering to [2] for a complete analysis. In the case of the gluino or neutralino, it is well–known that the CMSSM satisfies widely all the constraints imposed by flavor changing experiments [11]. Hence, this still holds true in this more general case, where we have shown that the sfermion mass matrices are still of the same size as in the CMSSM. This means then, that all possible mass insertions are always roughly two orders of magnitude bellow the required values to saturate flavor changing observables, (see second part of Ref [11]). Notice that this is true even for CP conserving flavor changing observables and the situation for the CP violating observables with chirality conserving operators, Eq. (6), is still much worse. Also chargino contributions can be comparable in general. This was the main subject of paper [2] where we showed the different constraints in the chirality conserving, $L$–$L$, and chirality changing, $L$–$R$, transitions. From [2] it is clear that chargino chirality changing transitions are **directly** constrained by the $b \to s\gamma$ decay to be more that three orders of magnitude smaller than the corresponding chirality conserving transitions. And finally, on the other side, we already showed in [2,6] that chirality conserving transitions were real to a very good approximation. These arguments allow us to discard measurable CP violation in both $\epsilon_K$ and indirect CP violation in the $B$ system.

Finally we have to consider also direct CP violation in non–leptonic $B$ decays. Essentially, the only difference with our discussion on indirect CP violation is the presence of the penguins. Once more, in the gluino case chirality conserving transitions are real to a very good approximation, and, in any case, well below the phenomenological bounds [11]. The chirality changing transitions on the other hand are suppressed by light quark masses, where we call light even the $b$ quark, and again below the bounds. Hence, our conclusion for the gluino is necessarily the same. So, we are left with chargino. $L$–$L$ transitions are real to a very good approximation, for the very same reasons used in the indirect CP violation case. And now the relation of $b \to s\gamma$ with the chirality changing penguins is even more transparent if possible. This completes the proof of our Theorem.

To conclude we would like to discuss the implications of our result in the search for supersymmetric CP violation. In the presence of large supersymmetric phases [1,4], the EDMs of the electron and the neutron must be very close to the experimental bounds. However, as we have shown in this letter, the presence of these phases is not enough to generate a sizeable contribution to $\varepsilon_K$, $\varepsilon'/\varepsilon$ or $B^0$ CP asymmetries. Here a completely new flavor structure in the soft breaking terms is required to get sizeables effects. In this sense, CP experiments in a supersymmetric theory are a direct probe on any additional flavor structure in the soft–breaking terms.

Hence, in the absence of new flavor structures, only pure chirality changing observables (EDMs or $b \to s\gamma$) or observables where, in any case, the chirality flip operators are relevant (*e.g.*, $b \to sl^+l^-$), can show the effects of new supersymmetric phases [7,2].

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